NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A MARKOV MODEL FOR MEASURING ARTILLERY FIRE SUPPORT EFFECTIVENESS

by

Dennis M. Guzik

September 1988

Thesis Advisor: Donald P. Gaver, Jr.

Approved for public release; distribution is unlimited

Unclassified

Security classification of this page

REPORT DOCUMENTATION PAGE

1a Report Security Classification Unclassified	1b Restrictive Markings
2a Security Classification Authority	3 Distribution/Availability of Report
2b Declassification Downgrading Schedule	Approved for public release; distribution is unlimited.
4 Performing Organization Report Number(s)	5 Monitoring Organization Report Number(s)
6a Name of Performing Organization Naval Postgraduate School — 6b Office Symbol *(if applicable)* 30	7a Name of Monitoring Organization Naval Postgraduate School
6c Address *(city, state, and ZIP code)* Monterey, CA 93943-5000	7b Address *(city, state, and ZIP code)* Monterey, CA 93943-5000
8a Name of Funding Sponsoring Organization — 8b Office Symbol *(if applicable)*	9 Procurement Instrument Identification Number
8c Address *(city, state, and ZIP code)*	10 Source of Funding Numbers: Program Element No — Project No — Task No — Work Unit Accession No

11 Title *(Include security classification)* A MARKOV MODEL FOR MEASURING ARTILLERY FIRE SUPPORT EFFECTIVENESS

12 Personal Author(s) Dennis M. Guzik

13a Type of Report	13b Time Covered	14 Date of Report *(year, month, day)*	15 Page Count
Master's Thesis	From To	September 1988	61

16 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes			18 Subject Terms *(continue on reverse if necessary and identify by block number)*
Field	Group	Subgroup	Artilley, Fire support, Markov, Counterbattery radar.

19 Abstract *(continue on reverse if necessary and identify by block number)*

This thesis presents a Markov model, which, given an indirect fire weapon system's parameters, yields measures of the weapon's effectiveness in providing fire support to a maneuver element. These parameters may be determined for a variety of different scenarios. Any indirect fire weapon system may be a candidate for evaluation. This model may be used in comparing alternative weapon systems for the role of direct support of a Marine Corps infantry battalion. The issue of light gun vs. heavy gun was the impetus for the study. The thesis also provides insight into the tactic of frequently moving an indirect fire weapon to avoid enemy detection, and possible subsequent attack.

20 Distribution/Availability of Abstract	21 Abstract Security Classification	
☒ unclassified/unlimited ☐ same as report ☐ DTIC users	Unclassified	
22a Name of Responsible Individual	**22b Telephone *(include Area code)***	**22c Office Symbol**
Donald P. Gaver, Jr.	(408) 646-2605	55Gv

DD FORM 1473,84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete — security classification of this page

Unclassified

A Markov Model for Measuring Artillery Fire Support Effectiveness

by

Dennis M. Guzik
Captain, United States Marine Corps
B.S., The Ohio State University, 1982

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1988

ABSTRACT

This thesis presents a Markov model, which, given an indirect fire weapon system's parameters, yields measures of the weapon's effectiveness in providing fire support to a maneuver element. These parameters may be determined for a variety of different scenarios. Any indirect fire weapon system may be a candidate for evaluation. This model may be used in comparing alternative weapon systems for the role of direct support of a Marine Corps infantry battalion. The issue of light gun vs. heavy gun was the impetus for the study. The thesis also provides insight into the tactic of frequently moving an indirect fire weapon to avoid enemy detection, and possible subsequent attack.

TABLE OF CONTENTS

I. INTRODUCTION

Field Artillery is known as the King of Battle for good reason. Great captains have relied upon its awesome firepower to change the course of battle for many years. It provides the commander with a force multiplier which is available day and night, has no weather constraints, and fires a wide variety of ammunition, both conventional and nuclear. In addition, it does not involve the sending of friendly personnel across enemy lines to deliver ordnance. Few other weapon systems can make these claims.

There is currently much debate in the artillery community as to the type of weapon system which is best suited for the direct support mission. Light weight, mobile howitzers or mortars provide flexibility and require little airplane or deck space, but do they have enough firepower to do the job? Larger, heavier howitzers, such as the M-198, have greater lethality, and an increased maximum range, but does this compensate for the lesser mobility caused by their size? Quite often it is the weapon system's characteristics which will determine the tactics used, and, ultimately, the level of support provided the maneuver element. This thesis will present a model of a firing battery, based on the weapon system characteristics of mobility, maximum range, and ammunition lethality.[1] These quantities will be represented by certain parameters in the model. The goals of this thesis are:

1. An analytical model which provides insight as to the effect the choice of weapon system has on the level of support provided to the supported maneuver element.
2. A look at battery survivability as a function of battlefield conditions, artillery tactics, and the weapon system choice.
3. A better understanding of the requirements for modeling indirect fire weapon systems.

Chapter 2 will detail the operations of a direct support field artillery battery. Chapter 3 will provide a Markovian model for this operation. Chapter 4 provides the results of applying the model in a simple example. Chapter 5 investigates the effect the decision to move has on the level of support provided by the battery, and on the battery's survivability. It will also show that by allowing the convenient assumption of exponential sojourn times in the states of the model, very little accuracy is lost, but much

1 Although it will be presented in terms of artillery, the model is applicable to any indirect fire weapon system.

is gained in mathematical tractability. Chapter 6 gives conclusions and recommendations for further study.

II. OPERATIONS OF A MARINE CORPS DIRECT SUPPORT ARTILLERY BATTERY

A. THE MISSION

Marine Corps artillery has the following responsibilities in support of the amphibious assault and subsequent operations ashore:

1. Providing fire in support of maneuver actions and as a part of the overall fire support effort to include:
   - Close support of maneuver units in combat.
   - Counterfire operations against enemy indirect fire systems.
   - Deep interdiction fire on enemy command posts, logistical installations, etc.
2. Provide fire support planning and coordination resources and facilities to all levels of force headquarters. [Ref. 1]

Each artillery unit is also given a tactical mission. It may be:

- Direct support (DS)
- General support
- Reinforcing
- General support reinforcing

Each tactical mission has its own order of priority by which it responds to requests for fire support. For example, a battery assigned a reinforcing mission will respond to a request for fire from the artillery unit it is reinforcing, before it does so for requests from another source. In the case of direct support, the battery will fire the requested fire missions from the supported unit before engaging other targets. In addition, for direct support, the zone of fire of the DS battery is the zone of action of the supported unit. This provides the supported unit commander with a dedicated fire support asset which he can base his maneuver plans upon. While any of these tactical artillery missions can be critical to the overall mission success, the area where there is the most debate, as to the appropriate weapon system to employ, is in the direct support role.

B. BATTERY ORGANIZATION

A Marine DS battery is composed of two firing platoons, and a headquarters platoon. Each firing platoon has four howitzers. The battery is commanded by a captain, and each platoon by a lieutenant. The battery is one of three firing batteries in a direct support artillery battalion. The battalion is commanded by a lieutenant colonel. The battalion provides direct support to an infantry regiment, while each of the batteries support one of the infantry regiment's battalions.

C. TACTICS

The firing battery may be brought into the battle by air, from the sea, or over land. The method used is determined by the tactical situation, terrain, transportation available, and the speed with which the fire support is needed. Once joined in battle, the most common method of moving the battery is over land. In the case of towed artillery, the towing is done by a "prime mover" suited for the load the weapons require.

The battery will occupy a firing position until an event occurs causing it to move. These events can be:

- The battery comes under attack.
- The current position does not allow the battery to support its maneuver element.
- The commander feels the threat of attack due to the time spent in the position, or the amount of firing done from the position, has reached an unacceptable level.

When any of the above events occur, the battery will displace to a new firing position, provided the force commander does not feel the current need for fire support outweighs the advantage the movement brings.

Determining the point at which the threat of attack is unacceptable is a difficult task. In many cases, a battery will have no indication that their position has been compromised until it comes under attack. In the past, rules concerning the time to move have been subjectively set by the commander. For example, a rule might be "displace the battery when you have been in position four hours, or after firing ten missions, which ever comes first".

The positions the battery occupy are, by doctrine, about one-third the maximum range of the weapon system behind the Foward Edge of the Battle Area (FEBA). The two firing platoons will occupy separate positions, typically 1 km. apart. The howitzers set up in the position to take advantage of cover and concealment present, and while not firing, work on improving it.

D. TARGETS

Targets may be acquired by many different means. The most common source of targets for a direct support artillery battery is by a Foward Observer (FO). FOs are battery personnel attached to the supported maneuver element. Their purpose is to detect and request fire on targets in the maneuver element's area of operation. They also help the maneuver element commander plan fire support. Targets may also be detected by countermortar/counter battery radar, aerial observers, and intelligence sources external to the artillery unit.

Targets are classified as to their degree of prearrangement. They may either be planned, or targets of opportunity. Planned targets have a high degree of prearrangement. Their firing data is prepared in advance, and they are either fired according to schedule, or on call. Targets of opportunity do not have firing data prepared in advance, hence a low degree of prearrangement. Because of this, it takes longer to engage a target of opportunity than a preplanned target. Targets are also assigned priorities, based on their potential threat to the supported unit.

E. THE FIRE MISSION

A fire mission is the act of bringing artillery rounds on a selected target. The processing of a fire mission is a complicated evolution, with many tasks to be accomplished concurrently. To begin, a FO may spot a target. He will then request fire on it by sending a "call for fire" to the battery Fire Direction Center (FDC). In this message he will provide such information as: the type of mission, the target location and description, and his preferred method of engaging the target.

The type of mission will usually be ***adjust fire, fire for effect, immediate smoke/suppression,*** or ***suppression of enemy air defense. Adjust fire*** means that one round of ammunition is fired by the battery, and a correction to bring the point of impact closer to the target is sent by the FO to the FDC. This continues until the last impact is within a specific distance from the target, based on the ammunition's lethality. At this point the entire battery engages the target. ***Fire for effect*** is used against targets whose location is accurately known, or when surprise or speed is a critical factor. In this case the entire battery will engage the target on the first volley. ***Immediate smoke/suppression*** is used if the supported unit is under attack, and speed of response is more critical than accuracy.

Suppression of enemy air defense is used to suppress enemy anti-aircraft weapons while friendly aircraft are operating in the area.

The FDC will simultaneously determine whether the battery is capable of engaging the target (i.e., a mission of higher priority is not being processed, the target is within range, and the ammunition is available), and if the target is in a restricted fire zone. The artillery battalion FDC and the Fire Support Coordination Center will monitor this message and determine if the target may be engaged more effectively by another fire support asset (e.g., naval gunfire, etc.), or, whether firing at this target will adversely effect other maneuver elements. If they are to allow the battery to fire the mission, nothing will be said. "Silence is consent" is the rule followed.

During this time the firing solution for the howitzers is computed. These data include which guns will fire, how many rounds they will fire, the type of round and fuze, the powder charge, deflection (direction), quadrant elevation, and the method of control. This information is transmitted to the gun sections, who then prepare their weapon for firing. Firing commences according to the controls established by the FDC.

When the determined number of rounds have been fired, the FDC will notify the FO of "rounds complete". The FO will either request additional rounds if the effect on the target has not been achieved, shift to another target if one is available, or end the mission. When the mission is ended, the FO will notify the FDC of his assesment of damage to the target.

The method of attacking a target is influenced by the results desired. This is usually to suppress, neutralize, or destroy a target. Suppressive fire is delivered to deny the enemy the opportunity to fire his weapon or maneuver freely. Accuracy is not as important as is the frequency with which the rounds impact. Neutralization fire attempts to degrade the enemy's combat efficiency. Surprise of the impacting rounds, and the ability to mass fire from several batteries, can be critical to the success of neutralization fires. Destructive fire attempts to make a target permanently ineffective. Accuracy and round lethality are important factors in a target destruction mission.

Although the fire support operation is complex, it is carried out quite efficiently. But, it can be improved. Having the proper weapon system for the job makes it much easier, and also more effective. Employing the tactics best suited for the weapon system used will enhance the support provided, while increasing the battery's survivability.

III. THE MODEL

A. INTRODUCTION

The Model is formulated so that it can represent a wide range of weapon types, from mortars to heavier self-propelled artillery. This was accomplished through the varying of certain model parameters. Different combat conditions, from low-intensity conflict through high-intensity conflict, can also be modeled by varying parameters in the model. Although a direct support artillery battery currently consists of two firing platoons, the model will consider it as an entity, to allow more flexibility in the comparison of alternative weapons and scenarios.

B. MODEL STATE SPACE

The battery will be modeled as an irreducible, recurrent Markov Chain, with state space as follows:

P_i The battery is in a firing position, available to support the maneuver element, and has fired i volleys since occupying this position. It has not been detected by the enemy.

Q_i The battery is in a firing position, available to support the maneuver element, and has fired i volleys since occupying this position. It has been detected by the enemy, but is unaware of it.

R_i The battery is in a firing position. It has been detected by the enemy, and it is aware of this. Because it is preparing to leave the position, it is not available to support the maneuver element.

U The battery is displacing to a new firing position. It is not available to support the maneuver element.

D The battery is "down" after sustaining an attack. It is not available to support the maneuver element.

C. MODEL PARAMETERS

The following are the parameters of the model.

J	Number of different target types the battery may engage.
λ_j	Mission (target) arrival rate of type j targets. $j = 1,2,...,J$
n_j	Number of volleys to fire at target of type j. $j = 1,2,...,J$
$\hat{p}_i$	Prob(detection from firing the i-th single round or volley)
ρ	Rate of response of the enemy once the battery is detected; $1/\rho$ is the mean time until the enemy attacks the battery, after detection.
δ	Rate at which the battery repairs itself after sustaining an attack; $1/\delta$ is the mean down time.
μ	Rate at which moves, between firing positions, are completed; $1/\mu$ is the mean time of a move.
θ	Rate at which the battery is attacked while moving; θ dt is the (approximate) probability that the battery is attacked at time t after the move has begun.
γ	Rate at which the battery determines its firing has caused detection.
α	Rate at which the battery leaves a position, once the decision to move has been made.
k	Number of rounds/volleys at which the battery will displace; k is a decision variable, the value of which is to be selected.

Table 1 on page 10 gives an interpretation of these parameters as a function of the weapon system and/or the scenario. Appendix A details additional considerations a user should make when estimating these parameters.

D. ASSUMPTIONS

The model makes the following assumptions:

1. Each time a battery occupies a new firing position, it is in an undetected state.
2. The primary source the enemy has for detecting the battery is countermortar/counterbattery radar. The detection of one round allows the accurate locating of the battery. In addition:

   Prob(detecting a single round) = Prob(detecting a volley fired simultaneously)

Other methods employed by the enemy for locating the battery also rely heavily on the battery's firing for detection.

3. The probability of detection is not a function of the number of rounds fired previously without detection, i.e.,

$$\hat{p}_i = \hat{p} \qquad i = 1,2, \ldots, k.$$

4. Missions (targets) of different types arrive according to a homogeneous Poisson process. The battery will fire a set number of volleys at each target type.
5. The battery will displace to a new firing position after firing a predetermined number of rounds/volleys, if it becomes aware of its own detection; see next; or if needed to maintain coverage of its supported element's zone of action.
6. If detected, there is a chance that the battery will become aware of this before it is attacked. It will then attempt to leave the position before the attack begins.
7. The battery cannot accept fire missions during its movement between firing positions.
8. If attacked in position, the battery will suffer casualties and battle damage, causing it to be unavailable to support the maneuver element for a period of time, or forever.

Table 1. **PHYSICAL INTERPRETATION OF THE MODEL PARAMETERS**

Parameter	high values of the parameter	low values of the parameter
J	Weapon system capable of engaging a variety of different target types due to its ammunition and its maximum range.	Weapon system capable of engagin limited number of different target ty because of ammunition and maxim range constraints.
λ_j $_{j=1,2,\ldots,J}$	High intensity conflict. Large weapon system maximum range.	low intensity conflict. Small weapon system maximum ra
n_j $_{j=1,2,\ldots,J}$	Small, less lethal rounds.	Large, more lethal rounds.
$\hat{p}_i$	An opponent with a very capable counterbattery locating system.	An opponent with a poor counterba locating system.
ρ	Very capable enemy. Battery located near the foward edge of the battle area. Battery one of few targets.	Relatively incapable enemy. Battery located far from the fow edge of the battle area. Battery one of many targets.
δ	Enemy response capacity limited. Battery in a fortified position.	Enemy response means capabl Battery in a vulnerable positio
μ	Mobile weapon system. Easily transported over the terrain.	Less mobile weapon system. Difficu transport.
θ	Enemy very capable of detecting and attacking a moving convoy.	Enemy with a lesser capability c detecting and attacking a movin convoy.
γ	Friendly means of detecting an impending attack is very good.	Friendly means of detecting an impending attack is poor.
α	Weapon system able to displace rapidly	Weapon system slow to displace

E. MODEL FORMULATION

Let:

$P_i(t)$ = Prob(being in state P_i at time t).

$Q_i(t)$ = Prob(being in state Q_i at time t).

$R_i(t)$ = Prob(being in state R_i at time t).

U(t) = Prob(being in state U at time t).

D(t) = Prob(being in state D at time t).

$p_j = 1 - (1 - \hat{p})^{n_j}$

$q_j = 1 - \hat{p}_i$

The rate equations are:

$$P_0(t+dt) = P_0(t)\left[\,1 - \sum_{j=1}^{J} \lambda_j\, dt\,\right] + U(t)\mu dt + D(t)\delta dt$$

$$P_i(t+dt) = P_i(t)\left[\,1 - \sum_{j=1}^{J} \lambda_j\, dt\,\right] + \sum_{l=0}^{i-1} P_l(t) \times \sum_{\substack{j=1 \\ s.t.\ l+n_j=i}}^{J} \lambda_j q_j\, dt \qquad i = 1,2,\ldots,k-1$$

$$P_k(t+dt) = P_k(t)\left[\,1 - \alpha\, dt\,\right] + \sum_{l=0}^{k-1} P_l(t) \times \sum_{\substack{j=1 \\ s.t.\ l+n_j\geq k}}^{J} \lambda_j q_j\, dt$$

$$Q_0(t) = 0$$

$$Q_i(t+dt) = Q_i(t)\left[\left(1 - \sum_{j=1}^{J} \lambda_j\, dt\right) \times (1 - \rho\, dt) \times (1 - \gamma\, dt)\right] +$$

$$\left[\sum_{l=0}^{i-1} P_l(t) \times \sum_{\substack{j=1 \\ s.t.\ l+n_j=i}}^{J} \lambda_j p_j\, dt\right] + \left[\sum_{l=1}^{i-1} Q_l(t) \times \sum_{\substack{j=1 \\ s.t.\ l+n_j=i}}^{J} \lambda_j\, dt\right] \qquad i = 1,2,\ldots,k-1$$

$$Q_k(t+dt) = Q_k(t)\,[(1-\alpha\,dt)\times(1-\rho\,dt)\times(1-\gamma\,dt)] +$$

$$\sum_{l=0}^{k-1} P_l(t) \times \sum_{\substack{j=1 \\ s.t.\ l+n_j \geq k}}^{J} \lambda_j p_j\,dt + \sum_{l=1}^{k-1} Q_l(t) \times \sum_{\substack{j=1 \\ s.t.\ l+n_j \geq k}}^{J} \lambda_j\,dt$$

$$R_i(t+dt) = R_i(t)\,[(1-\rho\,dt)\times(1-\alpha\,dt)] + Q_i(t)\,\gamma\,dt \qquad i = 1,2,\ldots,k-1$$

$$U(t+dt) = U(t)\,[(1-\mu\,dt)\times(1-\theta\,dt)] + P_k\,\alpha\,dt + Q_k(t)\,\alpha\,dt + \sum_{i=1}^{k} R_i(t)\,\alpha\,dt$$

$$D(t+dt) = D(t)\,[1-\delta\,dt] + \sum_{i=1}^{k} Q_i(t)\,\rho dt + U(t)\,\theta\,dt + \sum_{i=1}^{k} R_i(t)\,\rho\,dt$$

These equations may be solved for $P_i(t)$, $Q_i(t)$, $U(t)$, $R_i(t)$, and $D(t)$ [Ref. 2]. Because the Markov chain is irreducible and positive recurrent, the limiting distributions exist. Solving for them will allow the model to predict the proportion of time the battery will spend in a state where it can provide fire support to the maneuver element.

Let:

$\Pi(i) = \lim_{t\to\infty} P_i(t)$

$\Theta(i) = \lim_{t\to\infty} Q_i(t)$

$\Omega(i) = \lim_{t\to\infty} R_i(t)$

$\mathrm{U} = \lim_{t\to\infty} \mathrm{U(t)}$

$\mathrm{D} = \lim_{t\to\infty} \mathrm{D(t)}$

The steady state equations are:

$$\left[\sum_{j=1}^{J} \lambda_j q_j + \sum_{j=1}^{J} \lambda_j p_j\right] \Pi(0) = \mu\,U + \delta\,D$$

$$\left[\sum_{j=1}^{J}\lambda_j q_j + \sum_{j=1}^{J}\lambda_j p_j\right]\Pi(i) = \sum_{\substack{j=1 \\ s.t.\ i-n_j \geq 0}}^{J} \lambda_j q_j \Pi(i-n_j) \qquad i = 1,2,\ldots,k-1$$

$$\Pi(k) = \frac{1}{\alpha} \times \sum_{i=0}^{k-1} \sum_{\substack{j=1 \\ s.t.\ i+n_j \geq k}}^{J} \lambda_j q_j \Pi(i)$$

$$\left[\gamma + \rho + \sum_{j=1}^{J}\lambda_j\right]\Theta(i) = \sum_{\substack{j=1 \\ (i-n_j)\geq 0}}^{J} \lambda_j p_j \Pi(i-n_j) + \sum_{\substack{j=1 \\ (i-n_j)\geq 1}}^{J} \lambda_j \Theta(i-n_j) \qquad i = 1,2,\ldots,k-1$$

$$\Theta(k) = \frac{\displaystyle\sum_{\substack{j=1 \\ s.t.\ i-n_j \geq 0}}^{J} \lambda_j p_j \Pi(k-n_j) + \sum_{i=1}^{k-1} \sum_{\substack{j=1 \\ s.t.\ i+n_j \geq k}}^{J} \lambda_j \Theta(i)}{(\rho + \alpha + \gamma)}$$

where:

$$\Theta(0) = 0$$

$$\Omega(i) = \frac{\gamma}{\rho + \alpha} \times \Theta(i) \qquad i = 1,2,\ldots,k$$

$$U = \frac{\alpha}{\mu + \theta} \times [\, \Pi(k) + \Theta(k) + \sum_{i=1}^{k}\Omega(i) \,]$$

$$D = \frac{\theta}{\delta} \times U + \frac{\rho}{\delta} \times \sum_{i=1}^{k}\Theta(i) + \frac{\rho}{\delta} \times \sum_{i=1}^{k}\Omega(i)$$

For given values of J and n_j ,these equations can be solved as a function of $\Pi(0)$.

Let:

$\Pi(i)' = \Pi(i) \div \Pi(0)$

$\Theta(i)' = \Theta(i) \div \Pi(0)$

$\Omega(i)' = \Omega(i) \div \Pi(0)$

$D' = D \div \Pi(0)$

$U' = U \div \Pi(0)$

The sum of the limiting probabilities, over all states, must be one, therefore:

$$\sum_{i=0}^{k}\Pi(i) + \sum_{i=1}^{k}\Theta(i) + \sum_{i=1}^{k}\Omega(i) + U + D = 1$$

Then:

$$\Pi(0) + \sum_{i=1}^{k}\Pi(i)' \times \Pi(0) + \sum_{i=1}^{k}\Theta(i)' \times \Pi(0) + \sum_{i=1}^{k}\Omega(i)' \times \Pi(0) + D' \times \Pi(0) + U' \times \Pi(0) = 1.$$

Solving for $\Pi(0)$:

$$\Pi(0) = \frac{1}{1 + \sum_{i=1}^{k}\Pi(i)' + \sum_{i=1}^{k}\Theta(i)' + \sum_{i=1}^{k}\Omega(i)' + D' + U'}$$

Let: m(k) = The long run proportion of time the battery spends in a state in which support can be provided to the maneuver element. Then:

$$m(k) = \sum_{i=0}^{k}\Pi(i) + \sum_{i=1}^{k}\Theta(i)$$

$$= \Pi(0) + \sum_{i=1}^{k}\Pi(i)' \times \Pi(0) + \sum_{i=1}^{k}\Theta(i)' \times \Pi(0)$$

$$= \frac{1 + \sum_{i=1}^{k} \Pi(i)' + \sum_{i=1}^{k} \Theta(i)'}{1 + \sum_{i=1}^{k} \Pi(i)' + \sum_{i=1}^{k} \Theta(i)' + \sum_{i=1}^{k} \Omega(i)' + D' + U'}$$

The value of m(k) can be determined for each weapon system, allowing us to compare the percentage of total time each would be available, as a function of k.

IV. AN EXAMPLE

A. ALGEBRAIC SOLUTION

This chapter will provide an example of the model solving process. The parameters used are arbitrary. To simplify calculations, let:

J = 1

n_j = n = 1

$\lambda_j = \lambda$

p_j = p

q_j = q = 1 - p

This is equivalent to there being one type of target available for the battery to engage, and the firing of one volley will have the desired effect. A diagram of the model is in Figure 1 on page 17.

The steady state equations are:

For $\Pi(i)$:

$$\left[\sum_{j=1}^{J}\lambda_j q_j + \sum_{j=1}^{J}\lambda_j p_j\right] \Pi(i) = \sum_{\substack{j=1 \\ s.t.\ i-n_j \geq 0}}^{J} \lambda_j q_j \Pi(i-n_j) \qquad i = 1,2,\ldots,k-1$$

Which now simplifies to:

$$\lambda \times \Pi(i) = \lambda q \times \Pi(i-1) \qquad i = 1,2,\ldots,k-1$$

Solving for $\Pi(i)$:

$$\Pi(i) = q \times \Pi(i-1) \qquad i = 1,2,\ldots,k-1$$

This is solved in terms of $\Pi(0)$:

$$\Pi(i) = q^i \times \Pi(0) \qquad i = 1,2,\ldots,k-1$$

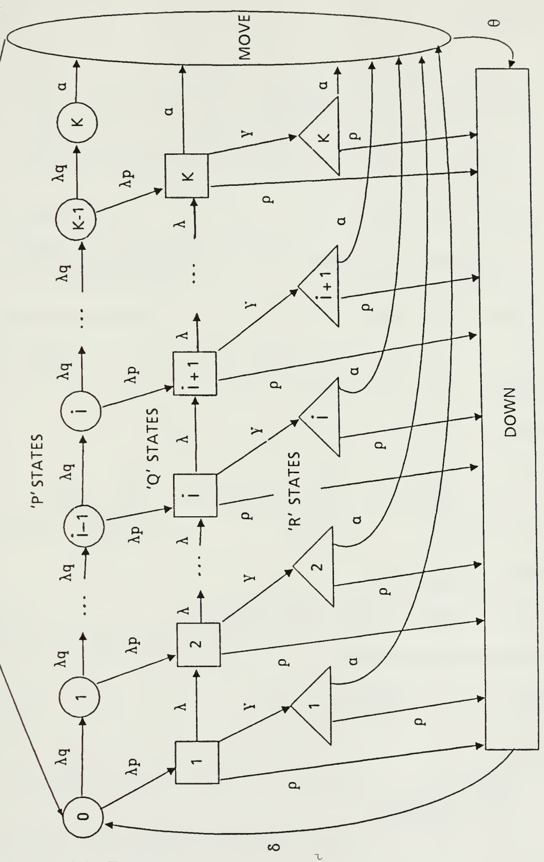

Figure 1. Diagram of the Example

For $\Pi(k)$:

$$\Pi(k) = \frac{1}{\alpha} \times \sum_{i=0}^{k-1} \sum_{\substack{j=1 \\ s.t.\ i+n_j \geq k}}^{J} \lambda_j q_j \Pi(i)$$

Which now simplifies to:

$$\Pi(k) = \frac{\lambda q}{\alpha} \times \Pi(k-1)$$

This is solved in terms of $\Pi(0)$:

$$\Pi(k) = \frac{\lambda q^k}{\alpha} \times \Pi(0) \qquad (4.1)$$

For $\Theta(i)$:

$$\Theta(0) = 0$$

$$\left[\gamma + \rho + \sum_{j=1}^{J} \lambda_j\right] \Theta(i) = \sum_{\substack{j=1 \\ s.t.\ (i-n_j) \geq 0}}^{J} \lambda_j p_j \Pi(i-n_j) + \sum_{\substack{j=1 \\ s.t.\ (i-n_j) \geq 1}}^{J} \lambda_j \Theta(i-n_j)$$

Which now simplifies to:

$$(\gamma + \rho + \lambda)\, \Theta(i) = \lambda p \Pi(i-1) + \lambda\, \Theta(i-1) \qquad i = 1,2,\ldots,k-1$$

This is solved in terms of $\Pi(0)$:

$$\Theta(i) = p \times C \times \left[\frac{q^i - C^i}{q - C}\right] \times \Pi(0) \qquad i = 1,2,\ldots,k-1$$

Where:

$$C = \frac{\lambda}{\gamma + \lambda + \rho}$$

For $\Theta(k)$:

$$\Theta(k) = \frac{\displaystyle\sum_{\substack{j=1 \\ s.t.\ i-n_j \geq 0}}^{J} \lambda_j p_j \Pi(k-n_j) + \sum_{i=1}^{k-1} \sum_{\substack{j=1 \\ s.t.\ i+n_j \geq k}}^{J} \lambda_j \Theta(i)}{(\gamma + \rho + \alpha)}$$

Which now simplifies to:

$$\Theta(k) = \frac{\lambda p \Pi(k-1) + \lambda \Theta(k-1)}{(\gamma + \rho + \alpha)}$$

This is solved in terms of $\Pi(0)$:

$$\Theta(k) = \frac{p\lambda}{\gamma + \alpha + \rho} \times \left\{ q^{k-1} + C \times \left[\frac{q^{k-1} - C^{k-1}}{q - C} \right] \right\} \times \Pi(0) \qquad (4.2)$$

For $\Omega(i)$:

$$\Omega(i) = \frac{\gamma}{(\alpha + \rho)} \times \Theta(i) \qquad i = 1,2,\ldots,k-1$$

For U :

$$U = \frac{\alpha}{\mu + \theta} \times \left[\Pi(k) + \Theta(k) + \sum_{i=1}^{k} \Omega(i) \right]$$

Where:

$$\sum_{i=1}^{k} \Omega(i) = \frac{\gamma p C}{(\alpha + \rho)(q - C)} \times \left[\frac{q - q^k}{p} - \frac{C - C^K}{1 - C} \right] + \frac{\gamma}{(\alpha + \rho)} \Theta(k)$$

And $\Pi(k)$ and $\Theta(k)$ are as determined in equations 4.1 and 4.2 .

For D :

$$D = \frac{\rho}{\delta} \times \sum_{i=1}^{k} \Theta(i) + \frac{\theta}{\delta} \times U + \frac{\rho}{\delta} \times \sum_{i=1}^{k} \Omega(i)$$

Where:

$$\sum_{i=1}^{k} \Theta(i) = \frac{(\alpha + \rho)}{\gamma} \times \sum_{i=1}^{k} \Omega(i)$$

Since the long run proportion of time spent in all the states must sum to one,

$$\sum_{i=0}^{k-1} \Pi(i) + \Pi(k) + \sum_{i=1}^{k-1} \Theta(i) + \Theta(k) + \sum_{i=1}^{k} \Omega(i) + U + D = 1.$$

Where:

$$\sum_{i=0}^{k-1} \Pi(i) = \sum_{i=0}^{k-1} q^i \times \Pi(0) = \left[\frac{1 - q^k}{1 - q} \right] \times \Pi(0)$$

and:

$$\sum_{i=1}^{k-1} \Theta(i) = \sum_{i=1}^{k-1} \left\{ p \times C \times \left[\frac{q^i - C^i}{q - C} \right] \right\} \times \Pi(0)$$

$$= \frac{p\,C}{q - C} \times \left[\frac{q - q^k}{1 - q} - \frac{C - C^k}{1 - C} \times \Pi(0) \right]$$

Therefore, letting:

$\Pi(i)' = \Pi\ (i) \div \Pi\ (0)$,

$\Theta(i)' = \Theta(i) \div \Pi\ (0)$,

$\Omega(i)' = \Omega(i) \div \Pi\ (0)$,

$D' = D \div \Pi\ (0)$,

$U' = U \div \Pi\ (0)$,

then:

$$\frac{1}{\Pi(0)} = \sum_{i=0}^{k-1}\Pi(i)' + \Pi(k) + \sum_{i=1}^{k-1}\Theta(i)' + \Theta(k) + \sum_{i=1}^{k}\Omega(i)' + D' + U'.$$

The long run proportion of time the battery is available to support the maneuver element, m(k), can be determined for different values of k.

$$m(k) = \frac{1}{\Pi(0)} \times \left\{ \sum_{i=0}^{k-1}\Pi(i) + \Pi(k) + \sum_{i=1}^{k-1}\Theta(i) + \Theta(k) \right\}$$

B. NUMERIC SOLUTION

As an example, let:

$p = 0.05$

$q = 0.95$

$\lambda = 4.$

$\rho = 2.$

$\delta = 0.1$

$\mu = 1.$

$\theta = 0.1$

$\alpha = 6.$

Solutions of m(k) for k from 1 to 15 are given in table Table 2 on page 22

Table 2. MODEL SOLUTION FOR M(K)

k	m(k)
1	0.1775
2	0.2323
3	0.2657
4	0.2872
5	0.3014
6	0.3113
7	0.3184
8	0.3238
9	0.3279
10	0.3312
11	0.3338
12	0.3360
13	0.3379
14	0.3395
15	0.3409

V. DETERMINING THE OPTIMUM MOVE STRATEGY

The number of rounds or volleys, k, is that which, when fired from the same firing position, will cause the battery to move to another position. It may be anticipated that if the battery stays in place for too long, measured in the number of rounds fired, it is discovered, and attacked, and hence out of action for a prolonged time. If it shoots and moves too soon its effectiveness is reduced by being unavailable during many moves. Some compromise may be advisable.

Typically, a firing battery will occupy a position for a predetermined time, or until it has fired a predetermined number of rounds. It is assumed that these are the main factors in the enemy locating, and subsequently, attacking the battery. Many types of locating systems are in use. These include radar, sound and flash ranging, aerial observation (visual and IR), as well as human reconnaissance. While not firing, the battery has a number of methods to avoid detection, including natural camouflage and IR reflecting nets. In most cases it is the battery's firing which gives its position away. It was for this reason that the model was based on the number of rounds fired, for determining when to move, and not the time in position.

In the initial construction of the model, it was thought that the value of k used in the comparison of weapon systems would be that k which produced the maximum value of m(k), i.e.,

$$k^* = \max_k \{ m(k) \}.$$

This implies the commander uses an optimal strategy for moving his firing unit. The values of m(k) would be found using the appropiate parameters for the weapon system and scenario considered. Graphs of m(k) vs k are in Figure 2 on page 24 ,for arbitrary sets of parameters.[2]

The most interesting result of this facet of the analysis was that for almost all reasonable combinations of parameters a pronounced optimum, not at an extreme point,

---

2 In each graph the parameters are as follows: $\lambda = 4, p = 0.05, \mu = 1, \rho = 2, \theta = 0.1, \delta = 0.1, \gamma = 0.1, \alpha = 6$; except: 1) The upper left graph, the solid line is for p = 0.01, and the dotted line is for p = 0.20; 2) The upper right graph, the solid line is for $\delta = 0.01$ and the dotted line is for $\delta = 1$; 3) The lower left graph, p = 0.20 and, the solid line is for $\theta = 0.01$ and and the dotted line is for $\theta = 1$; 4) The lower right graph, the solid line is for $\gamma = 0.05$ and the dotted line is for $\gamma = 2$.

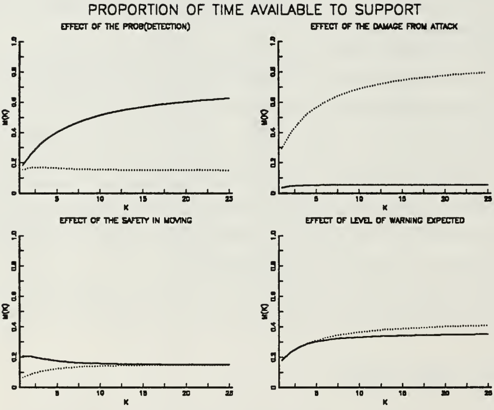

Figure 2. m(k) vs. k

rarely exists. Quite often, when a non-extreme optimal point exists, as in the dotted line in the upper left graph of Figure 2 on page 24 , the difference between the m(k^*) value obtained at the optimal point and that at very large values of k is very small. Using this method would then result in the decision being either: move after every mission, or do not base your decision to move on the number of rounds fired, in almost every case. It is of importance to note that the second instance, when m(k) is constantly increasing, does not that suggest that one should never leave the firing position. What it suggests is not to use the number of rounds fired from that position as the basis for decision. Instead, other factors, such as indications of an impending attack, time in position, or the need to maintain coverage of the supported unit's zone of action should be used as the basis for the decision.

When an optimal value exists between extreme points it is not always clear that this should be the decision rule. Since the model does not account for additional costs associated with moving the battery, such as the human fatigue and equipment wear, the small gain which frequent moving sometimes brings may be negated by these costs. When the optimum is significantly larger than the values at large k, then the rule would be of value, though this rarely occurs.

These results were viewed very skeptically at first. Artilley has, in the past, always used as a major factor in determining when to move a unit the number of rounds fired from its position. The determination of this number was usually left to the commander's judgement. It seemed intuitive that firing a great number of rounds from the same position would place the battery under increased risk of detection, and subsequent attack. In analyzing the model, the memoryless property of the exponentially distributed state sojourn times was investigated for its effect on m(k). A Fortran simulation was written [Appendix C] in which the Down, Moving, and Response time were made nonexponential, with the same mean the Markov model's states had.[3] As Figure 3 on page 27 shows, the results were very similar, especially at higher values of k. While this tended to disprove the possibility of the exponential sojourn times causing the lack of a non-extreme maximum it also showed that allowing the convenient assumption of exponential sojourn times did not detract from the model's accuracy very much. This assumption made the model much more mathematically tractable. Making the down time a constant, equal to the mean of the exponential down time, was also done with the

3 They were made the sum of five identical exponential distributions.

same simulation, with the thought that perhaps it was the variance of the sojourn times which caused the given results. The simulation showed that this was not the case.

Before accepting that the current method of deciding when to move the battery was perhaps in need of change, another method of determing k was attempted. By making the down state an absorbing state in the Markov model, the expected time the battery would be in states where it would provide fire support to the maneuver element before absorbtion (E(T)) could be determined. An example of this model formulation and solving process is detailed in Appendix B. It was felt that possibly this expected time would yield a definitive, non-extreme, maximum value, as a function of k. Figure 4 on page 28 is a display of two graphs of this expected time vs k. For most realistic combinations of parameters, the expected time function behaved as in the left graph of Figure 4 on page 28 . Rarely was there a maximum not at an extreme point. When it did exist, as in the dotted line of the same graph, it was rarely pronounced. The difference between the optimal value of the expected time, and the expected time at large values of k, was very small.[4]

While the results seemed counterintuitive, they were now supported by the two models. An explanation was sought as to why this may be the case. One reason for the results was that newer countermortar/counterbattery radars in use today do not need several rounds to locate a battery, as they did in the past. If an older system required N rounds to zero in on the battery, then moving before firing N rounds from the same position would make it impossible for their radar to locate the battery. The new radars need only a single round detected for an accurate locating of the battery. Because, like a battery firing, they too are subject to being located and attacked, these radars will not be active at all times. When they are active, if a battery fires into an area they are covering, the battery is detected and located, and presumably targeted for a future attack. If they are not active, then the battery has not been located, and it is as if it had not fired the last round (for detection purposes). Because it is not known who a future advesary will be, or what his radar doctrine will be, the model assumed a constant probability, p, of being detected. In actuality, the value of p may be a function of the amount of firing a battery is doing. If it is firing frequently, the the radar commander will feel that his becoming active will give him a high probability of detecting the battery,

---

4 Both graphs have as parameters: $\lambda = 4, p = 0.05, \rho = 2, \mu = 1, \alpha = 6$. The left graph has $\gamma = 0.05$, and, from lower to higher, $\theta = 1, 0.1, 0.01$. The right graph has $\theta = 0.1$, and, from lower to higher, $\gamma = 0, 1, 2$.

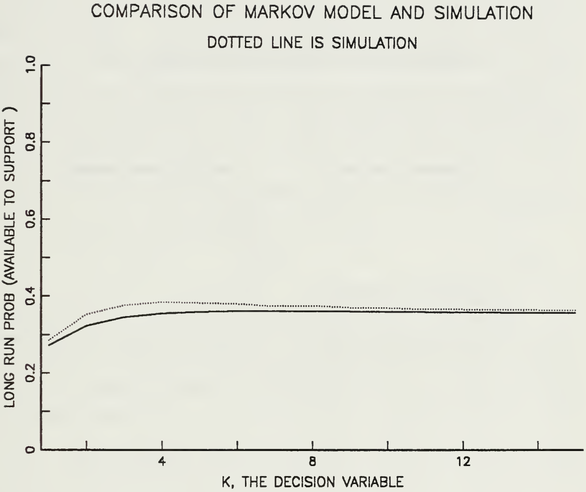

Figure 3. Comparison of m(k) from the Model and from a Simulation.

EXPECTED TIME IN SUPPORT UNTIL ATTACKED VS. K

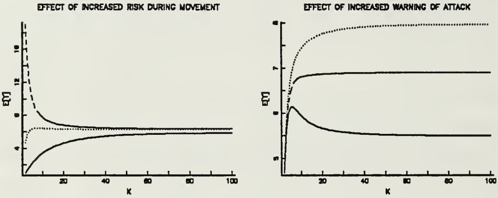

Figure 4. Expected Time Available to Support Until Attacked vs. k.

so p may increase with the firing rate of the battery. At the same time, though, the increased firing may be saturating the capabilities of the radar units, causing a decrease in p as the firing rate increases. A constant p reflects the fact that it is not known how the parameters will effect the probability of detection. This information should become known as a battle progresses.

Another factor in the shape of the m(k) and E(T) curves is the degree of uncertainty that exists with regard to a future attack on the battery. This is shown in the right graph of Figure 4 on page 28 . The model allows a probability, γdt , that once detected, the battery will know that it is detected in a time, dt. As this value increases, the decision to move becomes less a function of the number of rounds fired, and more a function of when the battery is informed of an attack. This also depends on the relative response rate of the attacking enemy. Knowing of an attack does little good if time isn't available to leave the position before it begins.

To solve the Markov model, and obtain a measure of effectiveness for competing weapon systems, a value of k is required. The calculation

$$k^* = \max_k \{ m(k) \}$$

will provide this value, if it exists, and it is not nearly equal the value of m(k) as k gets very large. When k is very small, the result would then be, basically, moving after each mission. When m(k) is constantly increasing, another method is needed, because the battery will not remain in a position for an indefinite time. In this case, the value of k used in the model should be:

$$k^* = \max_k \{ E(T) \},$$

if it exists, where:

E(T) = The expected time the battery will be available to support the maneuver element, before it is attacked.

An example of this calculation is in Appendix B. When m(k) and the expected support time are constantly increasing, the battery should move:

- to keep coverage of the supported unit's zone of action,
- if attacked,

- if warned of a suspected pending attack.

Then, let:

M_s = Time until battery moves to maintain support.

M_a = Time until battery moves because of an attack.

M_w = Time until battery moves because of a warning of an attack.

Then: M = Time the battery moves.

$$= \min\{M_s, M_a, M_w\}$$

Assume that M_s, M_a, M_w are independent and distributed exponentially, with means $1/u_s$, $1/u_a$, $1/u_w$, respectively. Then:

$$\bar{F}(M) = Prob(M > t) = e^{\{-(u_s + u_a + u_w)t\}}$$

It follows that:

$$E(M) = \int_0^{\infty} \bar{F}(M)\,dt = \frac{1}{u_s + u_a + u_w}.$$

The expected number of rounds to fire in this time is:

$$E(k) = E(M) \times \sum_{j=1}^{J} \lambda_j n_j,$$

which is the value of k to use in comparing weapon systems.

VI. CONCLUSIONS AND RECOMMENDATIONS

This chapter presents the conclusions arrived at during the model building effort and possible measures of effectiveness. In addition, recommendations for further study and a brief summary are given.

A. CONCLUSIONS

The initial model, formulated for the comparison of indirect fire weapon systems' effectiveness, was a fairly simple Markov chain. It consisted of k-1 undetected and detected states, a moving state, and a down state. The process transitioned from the (k-1)-st state to the moving state at the rate at which targets arrived (λ). This represented the battery leaving the firing position immediately after firing the k-th round. A k-th undetected and detected state was added to the model, with transitions to the moving state occuring at a rate different from the target arrival rate. This rate (α) was representative of the battery's need to perform certain tasks prior to moving. Since the time to complete this preparation is relatively short, the model's long run probabilities were changed very little by this modification. Solving for these probabilities was made more difficult, so a k-th state was not used in many future models when the goal was to determine the effect a specific model change had on the long run probabilites.

Since there is some risk to the battery in remaining in a firing position too long, but there is also a risk associated with moving, it is reasonable to expect that there would be an optimal movement decision variable (k). This would yield the largest value for the long run proportion of time the battery is available to provide fire support (m(k)). As was detailed in Chapter V, this was not always the case. In certain cases, such as when the probability of detection (p) was high, or the probability of being attacked while on the move was low, the model then determined that to maximize m(k) the battery should move after each mission. This conforms to what many in the artillery community feel will be the tactic in future conflicts. Artillery will be kept silent until the time when its firing will have the greatest impact on the battle. Only then will it fire, and then it will move.

In some instances, a maximum existed for m(k) which was not at an extreme, but it was never very pronounced. The difference between the maximum value of m(k), and its value at large values of k, was usually small. Since moving the battery entails additional costs not measured by this model (e.g., fatigue of the troops and wear on the

equipment), the small gain associated with moving frequently would probably not be worth these additional costs.

In some cases, the model predicted that the long run probabilities, m(k), will be increasing as k increases. This suggested that the decision to move not be based upon the number of rounds fired from the position. Rather, the decision should be based on such factors as the need to maintain coverage of the supported unit's zone of action, or because the battery has received some indication that its position has been compromised. To determine the effect advanced warning of an attack has, additional k states in which the battery has been detected and it knows this, were included in the model. These states had little effect on the m(k) values for most combinations of parameters. It did have an effect on the expected total time the battery is available to provide fire support before being attacked, E(T). As the probability of knowing of an attack increased, the maximum in the E(T) curve, if it existed not at an extreme point, became less pronounced. This should be expected, as there would be little reason to move because of fear of attack, when it is expected that there will be a warning before the attack. The presence of a pronounced maximum for E(T) was sensitive to the combined effects of some of the parameters. For example, while a high probability of knowing of an impending attack tended not to produce a clear maximum, as the average response time became shorter, the maximum became more pronounced. This should be expected, since knowing of an attack does little good if the response is so quick that the battery does not have the time to leave the position before the attack begins.

The lack of a pronounced maximum in the m(k) and E(T) curves may be due to improved systems for locating indirect fire weapons. If so, the tactic of moving based upon the number of rounds fired may be in need of change. But, the lack of a pronounced maximum may well result from assumptions made in formulating the model. Recommendations for further investigation of the effect of certain assumptions on the optimal move strategy are presented in the Recommendations section of this chapter.

1. Measures of Effectiveness from the Model

Given the value of a set of parameters representing a particular weapon system and scenario, and the value for the decision variable, k, determined using methods such as those described in Chapter V, the model can be solved for the long run proportion of time this weapon system is able to provide fire support to a maneuver element ($m(k^*)$). While this in itself is a measure of effectiveness for the support which that weapon system provides, other measures may also be determined. A weapon system may not be able to engage certain targets because of ammunition or range constraints. While

system may be available a greater proportion of time than is a competing system, the level of support it provides can be less, if the competing system does not have as great an ammunition or range constraint. A possible method of measuring fire support effectiveness would be the "gain" a maneuver element commander feels he gets from having a particular weapon system supporting him. To measure this gain, a value, V_j, must be assigned to each of the J target types. This value reflects the advantage a maneuver element commander feels is provided him by the battey's having the desired effect on target j. The expected gain per unit of time, could be computed as:

$$E[\,Gain\,] = m(k^*) \times \sum_{j=1}^{J} \lambda_j \times V_j. \qquad 60$$

Because J will depend heavily upon the weapon's maximum range, this will more accurately assess the impact of this parameter on the level of support provided.

This expected gain could also be used in comparing tactics. As an example, a commander may locate his firing unit further behind friendly lines, thus increasing its security, but decreasing the targets it can engage. By changing the model's parameters to reflect this, a comparison of the expected gain in each case would provide a measure of the effectiveness of this tactic.

The expected total time the battery was available to support the maneuver element, until it is attacked, can be determined as is demonstrated in Appendix B. This may prove to be a more important measure in certain circumstances, such as when replacements for attacked personnel and equipment are not available, or will take an extremely long time to acquire.

Expected costs can also be determined from the model.

$$E[\,Cost\,] = m(k^*) \times \sum_{j=1}^{J} \lambda_j \times n_j, \qquad 61$$

is the expected cost, in terms of the number of rounds of ammunition per unit of time which is needed to obtain the expected gain. This cost may then be transformed into logistic weight, and/or volume, whichever is more appropiate for the analysis.

B. RECOMMENDATIONS

Time did not permit the analysis of several factors impact on the model. These are presented as recommendations for further study.

1. The Movement Decision Strategy.

The problem of determining the optimal move strategy has been detailed in Chapters V and VI. A possible cause for the lack of a more pronounced optimum in the m(k) curve may be due to the assumption of a constant probability of detection, p. One method to investigate the effect of p on the values obtained for m(k), would be to make p an increasing function of the number of rounds fired from the position. This would represent an increasing hazard for the battery's staying in position. Another method would be to increase p during periods of relatively high firing rates and decrease p during periods of low firing rates. This would model the enemy's greater willingness to risk detection, from radiating his radar, because of his perceived increased probability of detecting a fired round.

If it appears that m(k)'s lack of a pronounced maximum is not due to the model, but accurately reflects current counterbattery locating system's capabilities, then moving a battery based on the number of rounds fired from a position may not be a correct tactic in many instances. The effect of time in the position, or ability to cover the supported maneuver element's zone of action, or even the level of intelligence the unit can expect to receive, may be the driving force in determining when to move.

2. Model Refinement.

The factors used in creating this model are not exhaustive. They were chosen because they were felt to be those with the greatest impact on the effectiveness of fire support a weapon system could provide. By modifying existing parameters, and including other factors, the model will more closely resemble the operations of a firing battery.

A possible modification would be to let target arrivals be a function of time, such as in a non-homogeneous poisson process. Perhaps this function of target arrivals may be represented by a Lanchester-type combat model.

The model can be ammended to reflect the effect of mechanical reliability on availability. This could be done as a function of the time spent moving, and the number of rounds fired, to model the effect these events have on equipment availability.

When target arrivals are very fast, or the weapon system's rate of fire is slow, or the number of rounds/volleys required to effect the target is large, there are bound to be targets arriving before previous missions have been completed. Since the rate of fire

and number of rounds required for effect are specified for a specific weapon system and target, the model can be changed to allow targets to join a queue. The order of firing, of the targets in the queue, will depend on the the priority assigned to each target. This will give an additional measure of a weapon system's parameters effect on fire support. (e.g., the average time a target spends in a queue, or the average length of a queue).

3. Other Uses for the Model

This model may be of use for other military systems. The radar systems, which attempt to locate a target, operate in a fashion similar to that of a firing battery. It actively emits radiation for a period, which while it provides benefits (target locating), it also makes it vulnerable to being located, and subsequently attacked. Radars usually do emit, for a period of time, then relocate. This model may be of help in evaluating radar systems and tactics.

C. SUMMARY

The purpose of this thesis is to provide those who must choose the artillery weapon system which the Corps will fight with in the next conflict, with a tool to better help them make this decision. It is not intended to provide a definitive answer to the question of light gun vs. heavy gun. It is intended to demonstrate, within the limitations of any probabilistic model, the effects of important weapon system parameters and environmental conditions on some measures of the level of support a weapon can provide. As with any model, the results are only as good as the assumptions used to formulate the model and the accuracy of the estimated parameters. Implementing the above recommendations should improve the model. While the decision maker must estimate the parameters, this also must be done if a simulation were to be used. When a decision maker states that he "feels" that a particular weapon system is more suited for a task, he is implicitly estimating these parameters also. The model is able to take these estimated parameters and show how their combinations effect fire support effectiveness. In addition, the effect of changes in parameters are easy to evaluate, especially when compared to performing a large scale simulation.

Deciding which weapon system to arm our forces with must include some attempt at predicting the future battlefield. While, predicting the future is, in general, risky it is much more so for combat operations. The cost of being incorrect is much higher, and the uncertainty greater. Nonetheless, it must be done. This model may be of some help.

APPENDIX A. CONSIDERATIONS FOR PARAMETER VALUES.

This appendix will give a little more insight into the physical considerations which should be made, in determining the parameters to be used, when implementing this model.

Parameter

J This parameter represents the number of different targets and missions which the firing battery may engage. As such, it reflects the combination of a specific target, e.g., a tank platoon, and a specific mission, e.g., "fire for effect". This is necessary because the result of artillery fire will have a different "value" to the Commander for different targets. Also, the same target, but engaged in a different mission will require the firing of a different amount of ammunition. This will lead to a different probability of detection, and a different expected cost, in terms of ammunition fired.

A weapon system's maximum range greatly effects J. A weapon with a relatively long maximum range will be able to engage targets which are out of range of a weapon with a shorter maximum range. The variety of ammunition a weapon system is capable of firing will also effect J. If there are targets which may be ranged by the battery, but whose ammunition has very little, if any, effect on the target, then that target should not be counted in J. It would be very unlikely that it would be assigned as a target to the battery.

The level of intensity of combat which the maneuver element is engaged in will effect J. It is assumed that combat with Warsaw Pact forces will provide a wider variety of target types, than would combat with a lesser developed nation. An additional concern is the means of locating a target. If the scenario depicted has little friendly targeting assets, than having the range and ammunition to attack a target is of little value if the target is never located.

λ_j Type j targets ($j = 1,...,J$), arrive at the battery at the rate λ_j. As such,

$$\frac{\lambda_j}{\sum_{j=1}^{J} \lambda_j}$$

is the probability that the next target to arrive will be of type j.

This rate is greatly effected by the intensity of the combat. It is also effected by the amount of other friendly fire support assets available. The greater the number of options available for fire support, the less frequent will be the assignment of targets to the battery.

n_j Target j will have n_j rounds/volleys fired at it, and, it is assumed that this will have the desired effect on the target. As an example, if current tables indicate that 24 rounds of a given caliber must be fired at a target, then n_j for an 8 gun battery would equal 3, (if the mission were fire for effect). If it were an adjust fire mission, an additional number of "adjusting rounds" should be added to n_j The major factors influencing n_j are the ammunition's lethality, the weapon's firing errors, and the target locating errors.

p_i The parameter p_i is the probability that the battery will be located when firing the i-th round/volley from the same position. While the model assumed this was constant, this need not be the case. The value assigned p_i should reflect the firing signature of the weapon, the rate of fire required by the combat conditions, and the quantity and quality of the enemy's indirect fire weapon locating systems. A relatively long maximum range can contribute to reducing the probability of being detected by non-radar methods, if the battery is positioned further behind the Foward Edge of the Battle Area (FEBA).

ρ The response rate of the enemy is modeled as ρ . As such, $1/\rho$ is the average time it takes for the enemy to attack the battery, once it has detected it.

This parameter has several factors affecting it. One is the importance the enemy places on attacking the battery. In a high intensity combat scenario, a nuclear-capable artillery battery will have a high priority for being attacked, once located. In a situation where the use of nuclear weapons is extremely unlikely, the fact that the weapon is capable of firing these munitions would have little effect. Also, if the battery's fire support is doing much harm to the enemy, then neutralizing the battery will assume a greater importance for the enemy.

The greater the number of similar priority targets (such as additional batteries) for the enemy to engage, the longer will be the average time until a specific battery is attacked. The response means available to the enemy is very important. If it is limited, then the response will probably take longer to occur. Air superiority should also be considered, since it will effect the number of options available to the enemy in attacking the battery.

The weapon's maximum range, and the tactic used to exploit it, contribute to ρ . A long maximum range allows the weapon system to locate further behind the FEBA, thus limiting the enemy's response options, while still maintaining coverage of the supported units zone of action.

α The parameter α represents the rate at which the battery leaves a position, once it has been determined that it should leave. Thus, the expected time required to prepare to move is $1/\alpha$. In addition,

$$\frac{\alpha}{\alpha + \rho}$$

is the probability that the battery will be attacked before it leaves a position, if it has been detected and it is aware of this detection. This parameter is also effected by the weapons mobility, and the terrain over which it must operate.

μ The parameter μ is the rate at which moves are completed. The averagetime to complete a move (and be prepared to fire at targets) is, then, $1/\mu$. The larger, more difficult to transport weapons will have a smaller value for μ, when compared to a lighter, more mobile system. Additional considerations are the terrain expected to be traversed, and the effect of weather on the terrain. Another concern may be the weapons size if it limits the number of firing positions which the battery may occupy. This could cause longer movement distances, thus, longer average move times.

θ The parameter θ represents the rate at which the enemy detects and attacks the battery when the battery is moving. As such, the probability that the battery is attacked before completing the move is:

$$\frac{\theta}{\theta + \mu}.$$

This parameter will depend on the signature the battery creates while moving, as well as the factors of enemy response, such as air superiority and available response means.

γ The rate at which the battery determines it has been detected is model as the parameter γ. The model is formulated such that

$$\frac{\gamma}{\gamma + \rho}$$

is the probability that, given the battery has been detected, it learns of this prior to it being attacked. The value of γ is largely a function of friendly capability to gather and pass to the battery, intelligence concerning enemy actions.

δ The parameter δ represents the rate at which the battery recovers from an attack. The mean time, from attack until the battery is prepared to accept fire missions, is $1/\delta$.

The value of this parameter will depend upon the severity of the attack, and the "hardness" of the battery at the time of attack. Of importance also, is the ability of the friendly unit to replace presonnel and equipment lost because of the attack.

APPENDIX B. EXAMPLE OF THE E(T) CALCULATION.

The calculation of the expected time a firing battery will be available to provide fire support until it is attacked is given for a simple example. This example will use k-1 states instead of k, for the reason given in Chapter VI, and will use the same assumptions as the example in Chapter IV, i.e., let:

$J = 1$

$n_j = n = 1$

$\lambda_j = \lambda$

$p_j = p$

$q_j = q$

This is equivalent to there being one type of target available for the battery to engage, and the firing of one volley will have the desired effect.

A. MODEL STATE SPACE

P_i The battery is in a firing position, available to support the maneuver element, and has fired i volleys since occupying this position. It has not been detected by the enemy.

Q_i The battery is in a firing position, available to support the maneuver element, and has fired i volleys since occupying this position. It has been detected by the enemy, but is unaware of it.

R_i The battery is in a firing position. It has been detected by the enemy, and it is aware of this. Because it is preparing to leave the position, it is not available to support the maneuver element.

U The battery is displacing to a new firing position. It is not available to support the maneuver element.

A The battery is "down" after sustaining an attack. It is not available to support the maneuver element. This is an absorbing state.

B. MODEL PARAMETERS

J	Number of different target types the battery may engage.
λ	Mission (target) arrival rate.
n	Number of volleys to fire at the target.
p	Prob(detection from firing a single round or volley)
ρ	Rate of response of the enemy once the battery is detected; $1/\rho$ is the mean time until the enemy attacks the battery, after detection.
μ	Rate at which moves, between firing positions, are completed; $1/\mu$ is the mean time of a move.
θ	Rate at which the battery is attacked while moving; θ dt is the (approximate) probability that the battery is attacked at time t after the move has begun.
γ	Rate at which the battery determines its firing has caused detection.
α	Rate at which the battery leaves a position, once the decision to move has been made.
k	Number of rounds/volleys at which the battery will displace; k is a decision variable, the value of which is to be selected.

C. MODEL FORMULATION

Let:

$P_i(t)$ = Prob(being in state P_i at time t).

$Q_i(t)$ = Prob(being in state Q_i at time t).

$R_i(t)$ = Prob(being in state R_i at time t).

U(t) = Prob(being in state U at time t).

A(t) = Prob(being in state A at time t).

The rate equations are:

$$P_i(t + dt) = P_i(t)\,(1 - \lambda\, dt) + P_{i-1}(t)\,\lambda\, q\, dt \qquad i = 1,2,\ldots,k-1$$

$$Q_0(t) = 0,$$

$$Q_i(t+dt) = Q_i(t)\,[(1 - \lambda\, dt)(1-\rho\, dt)(1-\gamma\, dt)] + P_{i-1}(t)\,\lambda\, p\, dt + Q_{i-1}(t)\,\lambda\, dt \qquad i = 1,2,\ldots,k-1$$

$$R_i(t+dt) = R_i(t)\,[(1-\rho\, dt)(1-\alpha\, dt)] + Q_i(t)\,\gamma\, dt \qquad i = 1,2,\ldots,k-1$$

$$U(t+dt) = U(t)\,[(1-\mu\, dt)(1-\theta\, dt)] + P_{k-1}(t)\,\lambda\, dt + Q_{k-1}(t)\,\lambda\, dt + \sum_{i=1}^{k-1} R_i(t)\,\alpha\, dt\ ,$$

$$A(t+dt) = A(t) + \sum_{i=1}^{k-1} Q_i(t)\,\rho\, dt + \sum_{i=1}^{k-1} R_i(t)\,\rho\, dt + U(t)\,\theta\, dt.$$

These yield the following first order differential equations:

$$P'_i(t) = -\lambda\, P_i(t) + P_{i-1}(t)\,\lambda\, q \qquad i = 1,2,\ldots,k-1$$

$$Q'_i(t) = -(\lambda + \gamma + \rho)\,Q_i(t) + Q_{i-1}(t)\,\lambda + P_{i-1}(t)\,\lambda\, p \qquad i = 1,2,\ldots,k-1$$

$$R'_i(t) = -(\alpha + \rho)\,R_i(t) + Q_i(t)\,\gamma \qquad i = 1,2,\ldots,k-1$$

$$U'(t) = -(\mu + \theta)\,U(t) + P_{k-1}(t)\,\lambda + Q_{k-1}(t)\,\lambda + \sum_{i=1}^{k-1} R_i(t)\,\alpha$$

$$A'(t) = \theta\, U(t) + \sum_{i=1}^{k-1} Q_i(t)\,\rho + \sum_{i=1}^{k-1} R_i(t)\,\rho$$

These differential equations can be solved by applying a Laplace transform of the form:

$$x_i(s) = \int_0^{\infty} e^{s\,t} \times X_i(t)\, dt.$$

This yields the following equations:

$$s\,p_i(s) \;=\; -\lambda\,p_i(s) \;+\; p_{i-1}(s)\,\lambda\,q \qquad i = 1,2,\ldots,k-1$$

$$s\,q_i(s) \;=\; -(\lambda + \gamma + \rho)\,q_i(s) \;+\; q_{i-1}(s)\,\lambda \;+\; p_{i-1}(s)\,\lambda\,p \qquad i = 1,2,\ldots,k-1$$

$$r_i(s) \;=\; \frac{\gamma}{s + \alpha + \rho} \times q_i(s) \qquad i = 1,2,\ldots,k-1$$

$$u(s) \;=\; \frac{\lambda}{s + \mu + \theta}\,[\,p_{k-1}(s) + q_{k-1}(s)\,] \;+\; \frac{\alpha}{s + \mu + \theta} \times \sum_{l=1}^{k-1} r_i(s),$$

$$a(s) \;=\; \frac{\theta}{s} \times u(s) \;+\; \sum_{i=1}^{k-1} q_i(t) \times \frac{\rho}{s} \;+\; \sum_{l=1}^{k-1} r_i(t) \times \frac{\rho}{s}.$$

These equations are now solved in terms of $p_0(s)$:

$$p_i(s) \;=\; \left(\frac{\lambda\,q}{s + \lambda}\right)^i p_0(s) \qquad i = 1,2,\ldots,k-1 \qquad (A.1)$$

$$q_i(s) \;=\; p\,C \times \left(\frac{\lambda\,q}{\lambda + s}\right)^i \times \left[\frac{1 - C^i}{1 - C}\right] p_0(s) \qquad i = 1,2,\ldots,k-1 \qquad (A.2)$$

where:

$$C \;=\; \frac{\lambda + s}{q\,(s + \lambda + \gamma + \rho)} \quad ,$$

$$r_i(s) \;=\; \frac{p\,\gamma\,C}{(s + \alpha + \rho)} \times \left(\frac{\lambda\,q}{\lambda + s}\right)^i \times \left[\frac{1 - C^i}{1 - C}\right] p_0(s) \qquad i = 1,2,\ldots,k-1$$

$$u(s) \;=\; \frac{\lambda}{(s + \mu + \theta)}\,[\,p_{k-1}(s) + q_{k-1}(s)\,] \;+\; \frac{\alpha}{(s + \mu + \theta)} \times \sum_{l=1}^{k-1} r_i(s). \qquad (A.3)$$

The values of $p_{k-1}(s)$ and $q_{k-1}(s)$ can be determined from equations A.1 and A.2, and:

$$\sum_{i=1}^{k-1} r_i(s) = \frac{\gamma}{(s + \alpha + \rho)} \times \sum_{i=1}^{k-1} q_i(s), \tag{A.4}$$

$$\sum_{i=1}^{k-1} q_i(s) = \frac{p(s+\lambda)}{(qB - (s+\lambda))} \left\{ \frac{\lambda q}{(s+\lambda)} \left[\frac{1 - \left(\frac{\lambda q}{(s+\lambda)}\right)^{k-1}}{1 - \left(\frac{\lambda q}{(s+\lambda)}\right)} \right] - \frac{\lambda}{B} \left[\frac{1 - \left(\frac{\lambda}{B}\right)^{k-1}}{1 - \left(\frac{\lambda}{B}\right)} \right] \right\} \tag{A.5}$$

and:

$$B = \lambda + s + \gamma + \rho.$$

$$a(s) = \frac{\theta}{s} u(s) + \sum_{i=1}^{k-1} q_i(s) \frac{\rho}{s} + \sum_{i=1}^{k-1} r_i(s) \frac{\rho}{s}$$

Where u(s) is determined in equation A.3, and

$$\sum_{i=1}^{k-1} q_i(s) \frac{\rho}{s} \quad \text{and} \quad \sum_{i=1}^{k-1} r_i(s) \frac{\rho}{s}$$

are determined in equations A.5, and A.4.

Let:

$$Z(t) = \begin{Bmatrix} 1 & \textit{if the battery can provide support at } t \\ 0 & \textit{Otherwise.} \end{Bmatrix}$$

E(T) = Expected total time the battery is available to provide fire support until attacked. Then:

$$E(T) = \int_0^\infty Prob(\, Z(t) = 1\,)\, dt$$

Since:

$$Prob(\, Z(t) = 1\,) = 1 - U(t) - \sum_{i=1}^{k-1} R_i(t) - A(t),$$

$$E(T) = \int_0^\infty \left(1 - U(t) - \sum_{i=1}^{k-1} R_i(t) - A(t) \right) dt$$

$$= \lim_{s\to 0} \left\{ \frac{1}{s} - u(s) - \sum_{i=1}^{k-1} r_i(s) - a(s) \right\}$$

$$= \lim_{s\to 0} \left\{ \sum_{i=0}^{k-1} p_i(s) + \sum_{i=1}^{k-1} q_i(s) \right\}$$

The summations of $p_i(s)$ and $q_i(s)$ have been solved in terms of $p_0(s)$. Since the sum of the probability of being in any state, over all states must be 1, $p_0(s)$ may be determined. Let:

$$p_i(s)' = p_i(s) \div p_0(s)$$
$$q_i(s)' = q_i(s) \div p_0(s)$$
$$r_i(s)' = r_i(s) \div p_0(s)$$
$$u(s)' = \mathrm{u(s)} \div p_0(s)$$
$$a(s)' = \mathrm{a(s)} \div p_0(s)$$

Then:

$$p_0(s) = \frac{1}{s \times \sum_{i=0}^{k-1} p_i(s)' + s \times \sum_{i=1}^{k-1} q_i(s)' + s \times \sum_{i=1}^{k-1} r_i(s)' + s\,u(s)' + s\,a(s)'},$$

and,

$$E(T) = \lim_{s \to 0} \left\{ \left(\sum_{i=0}^{k-1} p_i(s)' + \sum_{i=1}^{k-1} q_i(s)' \right) \times p_0(s) \right\}.$$

Solving for the expected total time the battery is available to support the maneuver element before absorbtion (attack), yields:

$$E(T) = \lim_{s \to 0} \frac{\sum_{i=0}^{k-1} p_i(s)' + \sum_{i=1}^{k-1} q_i(s)'}{s \times \sum_{i=0}^{k-1} p_i(s)' + s \times \sum_{i=1}^{k-1} q_i(s)' + s \times \sum_{i=1}^{k-1} r_i(s)' + s\,u(s)' + s\,a(s)'}$$

$$= \frac{\frac{1 - q^k}{p} + \frac{p\lambda}{q(\lambda + \gamma + \rho) - \lambda} \times \left\{ q \left[\frac{1 - q^{k-1}}{p} \right] - c \left[\frac{1 - c^{k-1}}{1 - c} \right] \right\}}{D},$$

where:

$$D = \left(\frac{\theta \lambda q^{k-1}}{\mu + \theta} \right) \times \left(1 + \frac{p c}{q} \left[\frac{1 - \left(\frac{c}{q}\right)^{k-1}}{1 - \left(\frac{c}{q}\right)} \right] \right) +$$

$$\left(\rho + \frac{\rho \gamma}{\alpha + \rho} + \frac{\alpha \theta \gamma}{(\alpha + \rho)(\mu + \theta)} \right) \times \frac{p\lambda}{q(\lambda + \gamma + \rho) - \lambda} \times \left\{ q \left[\frac{1 - q^{k-1}}{p} \right] - c \left[\frac{1 - c^{k-1}}{1 - c} \right] \right.$$

and:

$$c = \frac{\lambda}{\lambda + \gamma + \rho}.$$

APPENDIX C. VS FORTRAN SIMULATION CODE

```
*************************************************************************
* A PROGRAM TO SIMULATE THE EFFECT OF THE DECISION VARIABLE K
* (THE NUMBER OF ROUNDS FIRED FROM A POSITION BEFORE MOVING), ON
* THE LONG RUN PROPORTION OF TIME SPENT IN STATES WHERE FIRE
*SUPPORT MAY BE PROVIDED.
*
* KEY VARIABLES:
* K = NUMBER OF ROUNDS FIRED BEFORE MOVING.  DECISION VARIABLE
* P = PROBABILITY OF DETECTION FROM FIRING A SINGLE ROUND/VOLLEY
* N = NUMBER OF ROUNDS TO FIRE AT A TARGET
* TCSMO = TIME TO leave position (CSMO).  TCSMOU = MEAN TIME TO CSMO
* TMOVE = TIME TO MOVE.  TMOVEU = MEAN TIME TO MOVE.  DIST AS SUM OF
*         L EXPONENTIAL IID RANDOM VARIABLES.
* TTARG = TARGET INTERARRIVAL TIME.  TTARGU = MEAN TIME.
* TRESP = TIME TO RESPOND, FROM DETECTION.  TRESPU = MEAN TIME
*         DIST. AS SUM OF L IID EXPONENTIAL RANDOM VARIABLES
* TDOWN = TIME DOWN.  TDOWNU = MEAN TIME DOWN
*         DIST AS SUM OF L IID EXPONENTIAL RANDOM VARIABLES
* TIME = TOTAL TIME FOR THE EVENTS
* BADT = TIME SPENT IN THE MOVING AND DOWN STATES
* IT = NUMBER OF ITERATIONS
*
*************************************************************************

      PARAMETER (K=1, IA=12245, IB=33456, IC=35567, ID=45578,
     1IE=567893, IG=93765, IT=100, L=5)


      REAL TCSMO, TMOVE, TTARG(K), TRESP, TDOWN, T1(L), T2(L), T3(L)
     1,UZERO1(K), TIME(IT), BADT(IT)

      INTEGER DET

      CALL EXCMS('FILEDEF 11 DISK ARTYSIM OUT A1')

* SET THE PARAMETER VALUES
      P=0.05
      TCSMOU=10
      TMOVEU=60
      TDOWNU = 600.
      TRESPU = 30.
      TTARGU = 15.
      N = 1
* TMAX IS THE MINIMUM LENGTH OF TIME AN ITERATION WILL RUN
      TMAX = 100000.


      WRITE(11,*) 'K',K
      WRITE(11,*) 'CSMO MEAN',TCSMOU,'MOVE MEAN',TMOVEU
      WRITE(11,*) 'P',P,'DOWN MEAN',TDOWNU,'RESPONCE MEAN',TRESPU
      WRITE(11,*) 'TARGET ARRIV MEAN',TTARGU,'MAX TIME',TMAX,'IT',IT
```

```
      DO 888, J=1, IT

      TIME(J) = 0
      BADT(J) = 0

* GENERATE THE RANDOM NUMBERS
 5    CALL LEXPN(IA, TCSMO, 1, 1, 0)
      CALL LEXPN(IB, T1, L, 1, 0)
      CALL LEXPN(IC, TTARG, K, 1, 0)
      CALL LEXPN(ID, T2, L, 1, 0)
      CALL LEXPN(IE, T3, L, 1, 0)
      CALL LRND(IG, UZERO1, K, 1, 0)

      TMOVE=0
      TRESP=0
      TDOWN=0

      DO 3, I=1,L
         TMOVE=TMOVE + T1(I)
         TRESP=TRESP + T2(I)
 3       TDOWN=TDOWN + T3(I)

      DO 6, I = 1, K
  6      TTARG(I) = TTARGU*TTARG(I)
         TCSMO = TCSMOU*TCSMO
         TMOVE = TMOVEU*TMOVE/L
         TRESP = TRESPU*TRESP/L
         TDOWN = TDOWNU*TDOWN/L

      DET = 0
      PD = 1-(1-P)**N

* DETERMINE IF AND WHEN FIRING CAUSES A DETECTION
      DO 10, I = K/N, 1, -1
          IF(UZERO1(I).LE.PD) THEN

              DET = I

          END IF
 10   CONTINUE

* DET = 0 MEANS NO DETECTION OCCURRED FROM THIS FIRING POSITION
* THE FIRING UNIT WILL THEN DISPLACE.

      IF(DET.EQ.0) THEN

         DO 20, I= 1, K/N
 20      TIME(J) = TIME(J) + TTARG(I)

         TIME(J) = TIME(J) + TCSMO + TMOVE
         BADT(J) = BADT(J) + TMOVE
         GO TO 555

      END IF
```

```
* DETECTION OCCURS DURING FIRING
      IF(DET.NE.0) THEN
         RTIME = 0
         DO 30, I=DET+1, K/N
 30      RTIME = RTIME + TTARG(I)

         RTIME = RTIME + TCSMO

* DETERMINE IF THE RESPONCE TO THE FIRING OCCURS BEFORE THE FIRING
* UNIT DISPLACES.
         IF(RTIME.LT.TRESP) THEN

            DO 35, I = 1, DET
 35         TIME(J) = TIME(J) + TTARG(I)
            TIME(J) = TIME(J) + TMOVE + RTIME
            BADT(J) = BADT(J) + TMOVE
            GO TO 555

         END IF

* RESPONCE HAS OCCURRED BEFORE DISPLACING
         DO 40, I=1, DET
 40      TIME(J) = TIME(J) + TTARG(I)

         TIME(J) = TIME(J) + TRESP + TDOWN
         BADT(J) = BADT(J) + TDOWN

      END IF

 555  IF(TIME(J).LT.TMAX) THEN

         GO TO  5

       END IF

 888   CONTINUE

*  STATISTICAL SUMMARY AND OUTPUT
       PGOOD = 0

       DO 666, I=1, IT
 666   PGOOD = PGOOD + (TIME(I)-BADT(I))/TIME(I)

       SUM = 0

       DO 777, I = 1, IT
 777   SUM = SUM + (((TIME(I)-BADT(I))/TIME(I)) - PGOOD/IT)**2
       VAR = (1/(IT-1.))*SUM
       SD = VAR**(.5)

      WRITE(11,*) 'ESTIMATE OF PROPORTION IN SUP. STATE IS',PGOOD/IT
      PRINT *, 'ESTIMATE OF PROPORTION IN SUP. STATE IS',PGOOD/IT
      WRITE(11,*) 'ESTIMATE OF VAR  OF PROPORTION IN SUP. STATE IS',VAR
      PRINT *, 'ESTIMATE OF VAR OF PROPORTION IN SUP. STATE IS',VAR
      WRITE(11,*) 'ESTIMATE OF SD  OF PROPORTION IN SUP. STATE IS',SD
      PRINT *, 'ESTIMATE OF SD OF PROPORTION IN SUP. STATE IS',SD
```

```
WRITE(11,*) ' '
END
```

LIST OF REFERENCES

1. Department of the Navy, U.S. Marine Corps, Field Artillery Support, p. 1-1, FMFM 7-4, February 1981.

2. Taylor, H.M., and Karlin, S., An Introduction to Stochastic Modeling, p. 253, Academic Press, 1984.

BIBLIOGRAPHY

Center for Naval Analysis, The Field Artillery Simulation Model: A User's Guide (U), by Gubser, B.S., Confidential, September 1986.

Center for Naval Analysis Research Memorandum 87-28, The Marine Corps Artillery Structure Study (1986-1995): A Summary Report (U), by Kusek, P., Love, J.D., and Richardson, G.L., Confidential, February 1987.

Department of the Army, FM 6-20 Fire Support in Combined Arms Operations, Headquarters, Department of the Army, July 1984.

Department of the Army, FM 6-50 The Field Artillery Cannon Battery, Headquarters, Department of the Army, March 1983.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Commandant of the Marine Corps Code TE 06 Headquarters, U.S. Marine Corps Washington, D.C. 20360-0001	2
4.	Headquarters, U.S. Marine Corps POG - 34 Arlington, VA 20580	2
5.	Director of Analysis Support Attn: Dr. Branstein Warfighting Center (WF 13) MCCDC Quantico, VA 22134	1
6.	Director of the Fire Power Branch Attn: Major F. Sansone, USMC War Fighting Center MCCDC Quantico, VA 22134	1
7.	Professor Donald P. Gaver, Code 55Gv Department of Operations Research Naval Postgraduate School Monterey, CA 93943-5000	2
8.	LTC Bard Mansager, USA, Code 55Ma Department of Operations Research Naval Postgraduate School Monterey, CA 93943-5000	2
9.	Dr. Samuel H. Parry, Code 55Py Department of Operations Research Naval Postgraduate School Monterey, CA 93943-5000	1

10. Mr. E. Kovanic 2
Major Projects Office
Technical Directorate
Joint Tactical Command, Control, and Communications Agency
Ft. Monmouth, NJ 07703-5513

11. Captain Dennis M. Guzik, USMC 2
97 Grant Avenue
Totowa, NJ 07512